An Analytical Method for the Microwave Imaging of Cylindrically Stratified Media

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***Abstract* — An analytical method for the imaging of cylindrically stratified media is presented. The proposed method is based on the direct problem formulation of cylindrical dielectric objects using the concept of radial transmission line theory. This direct equation is then inverted to obtain a closed-form solution of the radially varying permittivity profile in terms of its value at the interface and the Hankel transform of the reflection coefficient data. A technique to obtain the value of the permittivity at the air-dielectric interface from the multi frequency scattering data is also presented here. A number of examples have been considered and a good agreement between the actual and the reconstructed profiles show that the proposed technique is suitable for both continuous and discontinuous profiles.**

I. Introduction

The imaging of underground objects for remote sensing applications, that of biological tissues for medical examination purposes, and of different materials for various industrial applications using low-intensity microwave radiation has gained much interest in recent years [1]-[3]. The imaging technique basically involves illuminating the object with an electromagnetic radiation, measuring the scattered field parameters, and obtaining an image of the distribution of the properties of the object from the measured field by using some reconstruction algorithm. The physical properties of the object, which interact directly with the microwave radiation are the permittivity and the conductivity. The reconstruction of permittivity profiles either qualitatively or quantitatively leads to the imaging of objects as different layers or materials can be distinguished by the difference in their permittivity values. For the remote sensing applications, the microwave also helps in detecting anti-personal mines which are non-metallic in nature and which can, otherwise, not be detected by a simple metal detector.

The development of an appropriate reconstruction algorithm for the determination of the permittivity profile is the most difficult part in the active microwave imaging process. The complexity arises because this is basically an inverse process and hence it is characterized by the nonlinearity and ill-posedness. In the past, many numerical methods have been used to solve these kind of problems, but generally they are computationally quite intense and are also more sensitive to ill-posedness [3]. We have recently proposed some analytical methods, based on field theory approach, to reconstruct the *continuously* inhomogeneous dielectric permittivity profiles in different coordinate systems [4]-[5]. The main advantage of our proposed technique has been its quasi-linearity and insensitivity to noise. The reconstruction of *stratified* permittivity profiles in cylindrical geometry on the other hand is useful for imaging many practical objects, whose boundaries coincide with the cylindrical coordinate surfaces. In the present contribution a transmission line approach has been used to reconstruct the permittivity profiles in stratified cylindrical geometries making use of the lowest order TM- illumination. It is worth noting here that the lowest order TM_{00} radial mode in the cylindrical geometry has neither E nor H in the direction of propagation and hence the classical transmission line theory can be used for the analysis in this case [6]. This radial transmission line approach helps in formulating the direct problem for layered or discontinuous cylindrical structures, which in turn leads to the reconstruction of the permittivity profiles of highly scattering cylindrical media in a more precise way. The analysis based on the transmission line approach also helps in determining the permittivity value at the air-dielectric interface, as will be shown in one of the following sections. Finally the radially stratified permittivity profile is reconstructed in terms of its value at the interface and the Fourier-Bessel transform of the reflection coefficient data.

II. Direct Problem Formulation for Cylindrically Layered Media

The geometry of cylindrically stratified media is shown in Fig. 1. This cylindrical inhomogeneous media is represented as a stack of N homogeneous layers and the permittivity in each layer is assumed to be constant. A TM_{00} cylindrical wave of a wave number k_0 is incident from the outer free-space and the reflection from this multi-layered media is measured at $\rho = b$.

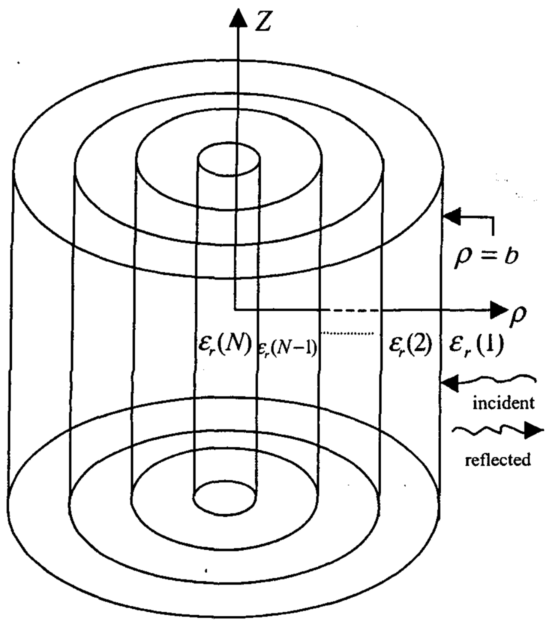

Fig. 1. Cylindrically stratified media

Meanwhile, as explained earlier, for the lowest order radial mode illumination, this N-layered cylindrical media can also be considered as N-cascaded radial transmission lines and the equivalent 2-port scattering parameter network can be used to represent each layer. The m^{th} 2-port network basically consists of a junction between two homogeneous layers of relative permittivities $\varepsilon_r(m-1)$ and $\varepsilon_r(m)$ and a section of radial transmission line of length $\Delta\rho$. The T matrix of each layer can be described as [7]:

$$[T] = [T_m^1]\,[T_m^2] \tag{1}$$

where,

$$[T_m^1] = \begin{bmatrix} \dfrac{\sqrt{\varepsilon_r(m-1)}+\sqrt{\varepsilon_r(m)}}{2\sqrt[4]{\varepsilon_r(m-1)\,\varepsilon_r(m)}} & \dfrac{\sqrt{\varepsilon_r(m-1)}-\sqrt{\varepsilon_r(m)}}{2\sqrt[4]{\varepsilon_r(m-1)\,\varepsilon_r(m)}} \\ \dfrac{\sqrt{\varepsilon_r(m-1)}-\sqrt{\varepsilon_r(m)}}{2\sqrt[4]{\varepsilon_r(m-1)\,\varepsilon_r(m)}} & \dfrac{\sqrt{\varepsilon_r(m-1)}+\sqrt{\varepsilon_r(m)}}{2\sqrt[4]{\varepsilon_r(m-1)\,\varepsilon_r(m)}} \end{bmatrix}; \tag{2}$$

is the transmission matrix of the junction between two homogeneous layers with relative permittivities $\varepsilon_r(m-1)$ and $\varepsilon_r(m)$, and

$$[T_m^2] = \begin{bmatrix} \dfrac{H_0^{(1)}\{k_m[b-(m-1)\Delta\rho]\}}{H_0^{(1)}[k_m(b-m\Delta\rho)]} & 0 \\ 0 & \dfrac{H_0^{(2)}\{k_m[b-(m-1)\Delta\rho]\}}{H_0^{(2)}[k_m(b-m\Delta\rho)]} \end{bmatrix} \tag{3}$$

is the transamission matrix of the mth line section, $H_n^{(p)}(k\rho)$ is the Hankel function of order n and type p,

$$k_m = 2\pi(f/c)\sqrt{\varepsilon_r(m)} = k_0\sqrt{\varepsilon_r(m)} \tag{4}$$

k_0 is the free space wave number, f is the frequency of the incident wave and c is the velocity of light. The total transmission matrix of the N-layered medium is obtained by multiplying the matrices describing each layer together [8]. Finally the reflection coefficient of the stratified cylindrical media, measured at the outer radius $\rho = b$ is given by:

$$\Gamma(k_0,\rho) = \frac{T_{21}}{T_{11}} \tag{5}$$

III. INVERSE SOLUTION

In the previous section, an inhomogeneous cylindrical medium is regarded as a stack of homogeneous layers and accordingly the direct problem has been formulated for the cylindrically layered media. If we assume that each layer in Fig. 1 is infinitely thin, then the length $\Delta\rho$ can be replaced by the differential length $d\rho$. The permittivity $\varepsilon_r(\rho)$ in this case can then be assumed to vary as a function of the radial distance ρ. Now suppose that in the region $0 < \rho \le b$, the reflection coefficients at points ρ and $\rho - d\rho$ are given by $\Gamma(k_0,\rho)$ and $\Gamma(k_0,\rho) - d\Gamma(k_0,\rho)$ respectively. As explained in the previous section, this small region of length $d\rho$ can be regarded as a transmission line and the permittivity can be assumed to be constant over this differential length. So, we have a radial transmission line of electrical length $\sqrt{\varepsilon_r(\rho)}d\rho$ followed by a scattering parameter network representing the junction between two layers of permittivity $\varepsilon_r(\rho)$ and $\varepsilon_r(\rho) - d\varepsilon_r(\rho)$ as shown below in Fig. 2:

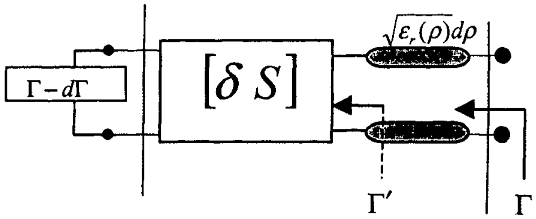

Fig. 2: The equivalent S-parameter network of a section of the cylindrical medium of length $d\rho$

In this figure, δS represents the S-matrix of the junction and Γ' is an intermediate variable. Using the microwave network theory [7] and (2) – (5), we arrive at the following equation:

$$\Gamma' = \frac{S_{11} + \Gamma - d\Gamma}{1 + \Gamma S_{11}} = \Gamma\left[1 + j\frac{4\,d\rho}{\pi\rho\, H_0^{(1)}(k\rho)\, H_0^{(2)}(k\rho)}\right] \quad (6)$$

where S_{11} is calculated from (2) and (5) with all the higher order terms of differentials like S_{11}^2, $d\Gamma^2$, $d\rho^2$ ..etc. being neglected. Now, (6) can be solved along with the value of S_{11} to obtain the following non-linear differential equation containing the reflection coefficient Γ and the radially varying permittivity profile $\varepsilon_r(\rho)$:

$$\frac{d\Gamma}{d\rho} + j\frac{4\Gamma}{\pi\,\rho\, H_0^{(1)}(k\rho)\, H_0^{(2)}(k\rho)} = \frac{1}{4\varepsilon_r(\rho)}\frac{d\varepsilon_r(\rho)}{d\rho}\left[1 - \Gamma^2\right] \quad (7)$$

It may be noted here that the structural form of (7) is somewhat similar to that presented in [4], although here a different approach has been used to derive this equation. The above nonlinear differential equation can be reformulated in terms of a virtual reflection coefficient $\hat{\Gamma}(k_0, \rho)$ and subsequently the modified equation can be solved using our previously proposed renormalization technique to obtain the radially varying permittivity profile in terms of the Hankel transform of the reflection coefficient data [5]. However, if there is a discontinuity at the air-dielectric interface i.e. at $\rho = b$, then the actual reflection coefficient data $\hat{\Gamma}$ should be modified to $\hat{\Gamma}_0$ [4]:

$$R_{01} = \frac{1 - \sqrt{\varepsilon_r(b)}}{1 + \sqrt{\varepsilon_r(b)}};\ \hat{\Gamma}_0(k_0) = \frac{\hat{\Gamma}(k_0) - R_{01}}{1 - R_{01}\,\hat{\Gamma}(k_0)} \quad (8)$$

where $\varepsilon_r(b)$ is the value of the permittivity at the air-dielectric interface. In the present contribution, we have also used a dummy time variable $t = (\rho / c)\sqrt{\varepsilon_r(\rho)}$ for a better accuracy and the modified reflection coefficient $\hat{\Gamma}_0(k_0)$ is transformed from the spectral domain to the virtual time domain $\hat{r}_0(t)$ using the Hankel transform. Finally, the inhomogeneous radially varying permittivity profile is reconstructed using the following equation:

$$\varepsilon_r(t) = \varepsilon_r(b)\ \exp\left[4\int_{t_b}^{t} G(\hat{r}_0(t'))dt'\right] \quad (9)$$

where $t_b = b\sqrt{\varepsilon_r(b)}$, and $G[\hat{r}_0(t)]$ gives the sampled values of the reflection coefficient data in the virtual time domain using a variable-resolution selective function [5]. The relationship between the virtual time variable t and the radial distance ρ is obtained using a numerical algorithm as described in [5].

IV. Determination Of The Permittivity at the Air-Dielectric Interface

As can be easily seen from (8) and (9), for the reconstruction of the radially inhomogeneous permittivity profile, we have to first determine $\varepsilon_r(b)$, the value of permittivity at the air-dielectric interface. For a discrete planar layered media, it has been shown that the parameters of different layers can be reconstructed successively so as to minimize the maximum of modulus of the reflection coefficient data for the remaining region in the frequency band of operation [8]. We have used the same principle here for the determination of $\varepsilon_r(b)$ for a cylindrically layered medium. The reflection coefficient of the region excluding the first air-dielectric interface can be derived using (2) and (8) to have the following form:

$$\Gamma_1(k_0) = \frac{[\Gamma(k_0) + 1]\ \sqrt{\varepsilon_r(b)}\ +\ [\Gamma(k_0) - 1]}{[\Gamma(k_0) + 1]\ \sqrt{\varepsilon_r(b)}\ -\ [\Gamma(k_0) - 1]} \quad (10)$$

The above function is optimized using the criterion $\max|\Gamma_1(k_0)| \rightarrow \min.$ over the given frequency band to obtain the value of $\varepsilon_r(b)$. This algorithm has been implemented using MATLAB and it just takes few seconds to obtain the correct value of the $\varepsilon_r(b)$ provided the reflection coefficient data is given over a wide frequency band.

V. RECONSTRUCTION EXAMPLES

We have reconstructed here both continuous and discontinuous permittivity profiles using our proposed method. Fig. 3 shows the actual and reconstructed permittivity for a step-like profile with a discontinuity at the outer air-dielectric interface ($\rho = 1$).

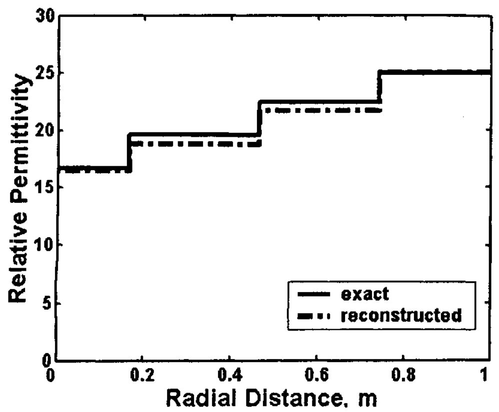

Fig. 3. Plot of actual and reconstructed profiles

As obvious from this curve, all the permittivity steps are reconstructed with a reasonable amount of accuracy. In this example, the reflection coefficient data in the range of 50 MHz to 10 GHz have been used and the value of permittivity at the air-dielectric interface has been determined using (10). This value of $\varepsilon_r(b)$ has then been used along with (8) and (9) to reconstruct the overall permittivity profile. The equations (1) to (5) have been used to generate the reflection coefficient data for this case. We have also reconstructed a continuous permittivity

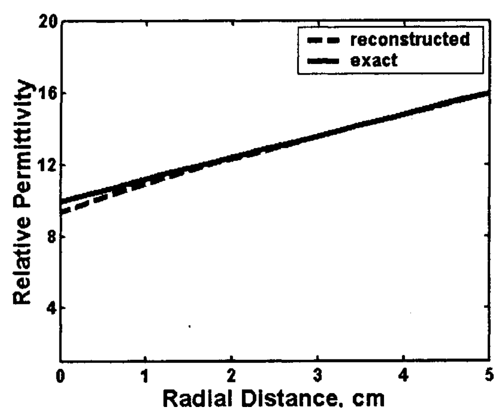

Fig. 4. Plot of exact and reconstructed profiles for a continuous case

profile as shown in Fig. 4. In this example also, a good proximity has been achieved between the exact and the reconstructed permittivity profiles. The reflection coefficient data in the range of 50 MHz to 15 GHz have been used for this continuous case and these data have been generated using (7).

VI. CONCLUSION

A technique for reconstructing the continuous and discontinuous (stratified) permittivity profiles of cylindrically layered media illuminated by a radial transmission line mode has been presented. An algorithm has also been proposed to determine the value of the permittivity at the air-dielectric interface for cylindrical objects. As a matter of fact, this algorithm can also be used to determine the permittivity for a small number of layers. A number of examples have been considered and a good agreement between the exact and the reconstructed profiles has been achieved in each case.

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